

Title page

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Title: Phya pa Chos kyi seng ge on argumentation by consequence (*thal 'gyur*) – The nature, function, and form of consequence statements

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Abstract

This paper presents the main aspects of the views of the Tibetan logician Phya pa Chos kyi seng ge (1109–1169) on argumentation “by consequence” (*thal 'gyur*, Skt. *prasaṅga*) based on his exposition of the topic in the fifth chapter of his *Tshad ma yid kyi mun sel* and on a parallel excursus in his commentary on Dharmakīrti’s *Pramāṇaviniścaya*. It aims at circumscribing primarily the nature and function of consequences (*thal 'gyur/thal ba*) for this author—in particular the distinction between “proving consequences” and “refuting consequences”—and the form prescribed for their enunciation in the context of debate. In addition to pointing out differences with the systems adopted by his predecessors, contemporaries and successors, the paper also discusses some of the similarities and differences between Phya pa’s understanding of argumentation by consequence and the notion of *reductio ad absurdum* in Western logic.

Key words

Phya pa Chos kyi seng ge; *thal 'gyur*; *prasaṅga*; *reductio ad absurdum*; argumentation; consequence; proof; refutation; *tshad ma*; Buddhist epistemology

Phya pa Chos kyi seng ge on argumentation by consequence (*thal 'gyur*) – The nature, function, and form of consequence statements

(Pascale Hugon, Austrian Academy of Sciences)

Introduction

The method of argumentation that consists in drawing unwanted consequences from premises the opponent subscribes to is attested in a variety of Indian philosophical systems.¹ In Buddhist philosophy, the question whether this method of argumentation termed *prasaṅga* (“consequence”)² should be used exclusively, or alongside proofs based on established premises (*svatantra*), was a debated point among Mādhyamika followers of Nāgārjuna. Far from being the only factor, this choice of argumentation was a significant criterion role in the Tibetan (or late Kashmirian³) doxographical distinction between *Svātantrika (*rang rgyud pa*) and *Prāsaṅgika (*thal 'gyur ba*) that was retrospectively applied to Indian authors. On the side of Buddhist epistemologists, Dharmakīrti addressed the question of the legitimacy of the use of *prasaṅga* in the third chapter of his *Pramāṇaviniścaya* (PVIN), where he elaborates on the discussion in *Pramāṇavārttika* 4.12. The legitimacy of *prasaṅga* resides, according to him, in the *prasaṅga*’s reverse formula, or *prasaṅgaviparyaya*, which qualifies as a proof based on established premises. Dharmottara and Prajñākaragupta prolonged the discussion. They disagreed in their commentaries on the conditions for the formulation of a reverse proof from a *prasaṅga*.⁴

This paper is the first part of a study that examines the developments brought to this topic by Tibetan thinkers in the early centuries of the Later Diffusion of Buddhism in Tibet (i.e., the 11th to 13th century).⁵ It gives an outline of the views of the famous logician Phya pa Chos kyi seng ge (1109–1169)⁶ on this subject, completing thereby my presentation of Phya pa’s theory of argumentation, the first part of which appeared in Hugon 2011. Phya pa resorts to argumentation by consequence when discussing various topics in his epistemological works as well as in his Madhyamaka works.⁷ But let us note already that the traditional ascription to this scholar of argumentation by “chains of consequences” (*thal phreng*), a type of argumentation that is widespread notably in later so-called *bsdus grwa* manuals used in the monastic curriculum, turned out not to be verified in the works that

¹ See Katsura 2005 for an overview and Watanabe 2013 for a discussion of the Sāṅkhya *āvāta* reasoning and a comparison with Dignāga and Dharmakīrti’s views on *prasaṅga*.

² The Sanskrit term *prasaṅga* is related to the verb *pra-saṅj* in the sense of “to follow, to result.” The Tibetan equivalent is *thal 'gyur* or *thal ba*. I deal in the conclusion with the question whether *thal 'gyur*, as understood by Phya pa, is comparable with the type of reasoning known as *reductio ad absurdum* in Western logic. While keeping this question open for now, I retain the Sanskrit term when speaking of the views of Indian authors and adopt the connotation-neutral rendering “consequence” to translate the Tibetan equivalent.

³ See Dreyfus&Tsering 2009(2010), especially p. 395.

⁴ For a detailed study see Iwata 1993. On this topic see also Tani 1992 and Iwata 1997.

⁵ Forthcoming parts of this study will address the following points: In the second part I examine in more details Phya pa’s basic classification of consequences and specific issues pertaining notably to the ontological nature of the subject and the logical reason. Part III goes into the details of the discussion about consequences with unestablished subjects, confronting rNgog Blo ldan shes rab and Phya pa’s views. In the fourth part I examine another division of consequences proposed by Phya pa, based on the type of logical reason involved. I also take up in this context the parallel (but diverging) division proposed by his successors, and attempt to solve the conundrum pertaining to Sa skya Paṅḍita’s version of this division in the RIGS GTER. In part V I look at Phya pa’s interpretation of the *prasaṅga*-passage in Dharmakīrti’s *Pramāṇaviniścaya* (PVIN). I relate this interpretation to the model of *thal 'gyur* he presents in the MUN SEL and the excursus in the 'OD ZER, and compare it to the commentary on the same passage by gTsang nag pa. Part VI takes up Phya pa’s Madhyamaka texts. I discuss on the one hand illustrations of consequences and their characterization in these texts, and on the other hand the exact nature of Phya pa’s criticism against followers of Candrakīrti whose arguments pertaining to emptiness rely on consequences alone. In the last part (VII) I present a critical edition of the relevant passages of Phya pa’s texts together with an English translation.

⁶ See Tauscher 2009–2010 for a recapitulation of the information available on his life and works drawing notably from van der Kuijp 1978 and 1983.

⁷ See Tauscher 2003 and 2009–2010 for a preliminary investigation of the latter.

have now become available in the *bKa' gdams gsung 'bum* collection.⁸ This paper leaves aside for the most part Phya pa's own use of argumentation by consequence to focus on his *theory*, as it is expressed in two of his epistemological treatises: the *Tshad ma yid kyi mun sel* (MUN SEL 83a2–95b5) and his large commentary on the PVIN ('OD ZER 145a2–149a5).⁹ It aims at circumscribing in particular the nature and function of consequences (*thal 'gyur/thal ba*) for this author, and the form prescribed for their enunciation in the context of debate. As usual in this kind of literature, Phya pa's discussion of the topic is not designed as a progressive introduction for beginners. The various elements of his discussion follow an internal logical order, but not a pedagogical one. Notably, the definition of a consequence is addressed in the very last section of his excursuses. My presentation will therefore not mirror the linear order of the source text.

The topic of argumentation by consequence was already discussed by Phya pa's Tibetan predecessors. In particular, a number of key-issues dealt with in MUN SEL (87b9ff) and 'OD ZER (146b4–148a4) were taken up by rNgog Blo ldan shes rab (1059–1109; hereafter rNgog Lo),¹⁰ as attested in his explanation of the difficult points of the *Pramāṇaviniścaya* (DKA' GNAS).¹¹ Also, Phya pa's refutations of the views of other thinkers introduce positions and alternative definitions of Tibetan origin. Some of them can be recognized as rNgog Lo's views. The proponents of some other views are nominally identified in the TSHAD BSDUS¹² as students of rNgog Lo or authors contemporary with Phya pa. Apart from this evidence, lack of materials does not allow us to conclusively assess to what extent the elements of Phya pa's presentation build on earlier presentations or constitute an original contribution.

The discussion of consequences in Phya pa's works goes a step further than Dharmakīrti's treatment of the topic and the discussion found in rNgog Lo's DKA' GNAS by engaging in the methodical classification of any argumentative statement of the form “Because S is P, it follows that it is Q.”¹³ Beside the distinction between genuine (*rnal ma*)¹⁴ and fallacious (*ltar snang*) consequences, Phya pa distinguishes among genuine consequences those that induce a reverse formula that amounts to a proof based on a correct (i.e., triply characterized) logical reason, and those that do not. The former are also meticulously classified based on the type of logical reason involved and according to whether the logical reason of the reverse formula is of the same type or of a different type.¹⁵ These constitutive elements of Phya pa's discussion of consequences are found also in the works of his disciples, notably gTsang nag pa brTson 'grus seng ge (?–after 1195), and mTshur ston gzhon nu seng ge (ca. 1150–1210).¹⁶ They are attested also in various works of this early period (I will notably mention the TSHAD BSDUS and the GSAL BYED by gTsang pa drug po rDo rje 'od zer¹⁷). They are

⁸ See Hugon 2008: 91–92. In such “chains” (*phreng*), the conclusion that follows from a first argument by consequence, if accepted, becomes the premise for a second consequence, etc. In the event of an objection to the first consequence, the argumentation proceeds to draw conclusions from there.

⁹ We may note here that even though the presentations in MUN SEL and 'OD ZER are parallel on most points, there is no evidence of literal re-use. In the second work the excursus on *thal 'gyur* precedes the word-commentary on the *prasaṅga*-passage of the PVIN ('OD ZER 149a5–150b3).

¹⁰ On his life and works see Kramer 2007.

¹¹ See DKA' GNAS 381–393. The excursus on consequence takes place in the context of the commentary on the *prasaṅga*-passage of the PVIN.

¹² On this work see van der Kuijp 2003.

¹³ For simplicity's sake, I refer here to the relation between the subject S and the properties P and Q with the English verb “is.” In Tibetan, both *yin* and *yod* occur. Note also that the properties can be negative or may be ascribed to S negatively (see the example in §1). The negation is sometimes attached to the property, sometimes to the verb (*min*, *med*). Whether the negation is internal (S is notP) or external (S is not P, i.e., not [S is P]) is only relevant when the subject is unestablished (cf. n. 73).

¹⁴ Unlike some of his successors Phya pa does not speak of such consequences as “correct” (*yang dag*), probably to avoid implying that they are “correct” in the same way proofs relying on true and established premises are.

¹⁵ The types of logical reasons under consideration are the three main types distinguished by Dharmakīrti—essential property, effect, non-apprehension—and the subtypes of the third of these.

¹⁶ gTsang nag pa's excursus on *thal 'gyur* is found in BSDUS PA 158a7–164b8. A parallel discussion is found in mTshur ston's SGRON MA 62b7–67a6.

¹⁷ The GSAL BYED postdates the compositions of Phya pa's direct disciples. It includes indeed numerous attributions of positions to Phya pa, gTsang nag pa and Dan 'bag pa. The identity of its author, identified as

included also in the presentation of a Tibetan view that Sa skya Paṇḍita (1182–1251) criticizes in his RIGS GTER. Before Phya pa’s texts became available, Onoda (1986) attempted to approximate Phya pa Chos kyi seng ge’s views based on gTsang nag pa’s text and this passage of the RIGS GTER. But although commentators of the RIGS GTER such as Glo bo mkhan chen (1456–1532) point to “Master Phya pa, etc.” to identify Sa skya Paṇḍita’s target, as Onoda rightly noted, there is a discrepancy between the number of divisions of consequences based on the type of logical reason attributed to the opponent in the RIGS GTER and the number of divisions ascribed to Phya pa by Śākya mchog ldan (1428–1507). Onoda suggested that Sa skya Paṇḍita was referring here not to the views of Phya pa himself, but to those of one of his disciples who also held a position different from that of gTsang nag pa. The texts now at our disposal confirm that Phya pa’s presentation was not adopted unanimously and thoroughly by his successors; I will point out some of the major points of dissension in the course of my inquiry.

1. The nature of “consequences” – consequences as a type of statement in a disputation

The place of the topic within the overall exposition of the MUN SEL clearly reveals what the nature of a consequence is for Phya pa:

- 1 In general, division of mind (*spyir blo tsam gyi dbye ba*)
- 2 In particular, establishment of mind consisting in valid cognition (*bye brag du tshad ma’i blo gtan la dbab pa*)
 - 21 Generic definition [of valid cognition] (*spyi’i mtshan nyid*)
 - 22 Division (*rab du dbye ba*)
 - 23 Perception (*mngon sum*)

Chap. 4 – Determination of inference (*rjes su dpag pa rnam par nges pa*)

24 Specific establishment of inference (*rjes su dpag pa bye brag du gtan la dbab pa*)

- 241 Definition of inference (*rjes dpag gi mtshan nyid*)
- 242 Consideration of correct and fallacious logical reasons (*gtan tshigs dang gtan tshigs ltar snang bsam pa*)
- 243 The nature of the probandum, which is the object to which the capacity of the logical reason applies (*rtags kyi nus yul bsgrub bya’i rang bzhin*)

Chap. 5 – Determination of the way to voice a statement in a debate (*rtsod pa’i ngag gi tshul rnam par nges pa*)

- 244 Division between proof statements and refutation statements (*bsgrub pa dang sun ’byin pa’i ngag gi dbye ba*)
- 245 The way to present these proof statements and refutation statements as consequences (*sgrub pa dang sun ’byin pa’i ngag de thal ’gyur du dgod pa’i tshul/brjod pa’i tshul*) (83a1–95b5)

The notion of “consequence” dealt with in section 245 of the MUN SEL refers to a type of formulation (*dgod pa’i tshul/brjod pa’i tshul*) that can be adopted for the statement of a proof or of a refutation (*sgrub pa dang sun ’byin pa’i ngag*). As such, consequences, or more precisely consequence statements (*thal ba’i ngag*), are parts of the statements made in a disputation (*rtsod pa’i ngag*).

1.1 Actual consequence statements and reference to consequence statements

Before we go any further, it is necessary to distinguish instances of consequence statements — what participants in the debate do express or should express — from the way Phya pa refers to these instances. Indeed, Phya pa mainly refers to instances of consequence statements using the form:

- (a) Because S is P, it follows that it is Q

Ex: Because there is no fire on the pass, it follows that there is no smoke (*la la me med pa’i phyir du ba med par thal*)

But such a statement is not formally speaking a correct consequence statement according to Phya

“gTsang pa drug po” and “the Buddhist monk rDo rje ’od zer” in the colophon, requires further investigation. One should consider in addition the identification of the author’s Bla ma as “dge ba’i bshes gnyen gnyal pa zhang tshes,” i.e., the kalyānamitra gNyal pa zhang tshes. A “gTsang pa drug po” is listed among the disciples of Ko brag pa in the DEB SNGON (853), and according to Smith (2001: 180), the “nine sons of gNyal zhig” was a disciple-line of Ko brag pa (1182–1253). gTsang pa drug po could thus be identical with gTsang pa gru gu, who is listed in the first group of the nine “sons” of gNyal zhig ’Jam pa’i rdo rje in the DEB SNGON (407; cf. Sparham 1996: 23). Bu ston informs us that gTsang pa gru gu was contemporary with gZhon nu brtson ’grus, the sixth abbot of Zhwa lu (Sparham 1996: 23). And in the biography of bCom ldan ral gri by bSam gtan bzang po, a “gTsang pa drug po” is said to have been a teacher of Ral gri in 1239.

pa’s own rules.¹⁸ As I discuss in §3.3, it is not acceptable in particular because it makes explicit the conclusion to be derived from the premises, i.e., “it follows that...” A standard correct consequence statement (we will see in §3.2 that a number of variations can apply) must rather be of the form:

(b) What is P is Q; [according to you] S is P

Ex: Where there is no fire there is no smoke; you accept that there is no fire on the pass, hence... (*gang me med pa na du ba med de chu klung bzhin la la me med khas blangs pas zhes*)

Thus all formulations of type (a) must be taken as a meta-linguistic way to refer to consequence statements of type (b), and not as *bona fide* consequence statements. I adopt for clarity’s sake a formulation of type (a) for all the examples given in this paper (except in §3.2), precisely because it makes explicit the conclusion to be derived and preserves the mark of a consequence, namely the expression “it follows that” (*thal*), which is absent in formulations of type (b).

1.2 Two alternative types of formulation: consequence and direct proof

According to the title of the relevant section of Phya pa’s MUN SEL, a proponent who puts forward a proof and a respondent who puts forward a refutation of the proponent’s proof have the choice between two types of formulation. I concentrate here on the case of proof.¹⁹ Let us consider the stock-example of a proponent who wishes to prove to his opponent that there is fire on a pass where smoke can be observed. Such a proof can take two forms:

	Reference to the statement	Actual standard formulation in a debate
(1) <i>rang rgyud</i>	Since there is smoke on the pass, there is fire. ²⁰	Where there is smoke there is fire; there is smoke on the pass, hence...
(2) <i>thal ’gyur</i>	Because there is [according to you] no fire on the pass, it follows that there is no smoke.	Where there is no fire there is no smoke; [you accept that] there is no fire on the pass, hence...

The first type of formulation (1) is a standard proof statement presenting the opponent with a triply characterized reason (here: “fire”), which enables the latter to recall the characteristics of the logical reason and perform an inference-for-oneself.²¹ In the second case (2), the proponent puts forward a premise that is here merely accepted by the opponent (“there is no fire on the pass”), which, together with the premise of pervasion (“were there is no fire there is no smoke”) that in this case is truly established, but in general must hold at least according to the opponent for the consequence to be genuine, leads the latter in the first place to the absurd conclusion that “there is no smoke.”

The first type of formulation is termed *rang rgyud* (reflex of the Sanskrit *svatantra*).²² The second type of formulation is called *thal ’gyur* or *thal ba* (terms that translate the Sanskrit *prasaṅga*), referring to

¹⁸ It may be acceptable for other authors. See notably Śākya mchog ldan’s explanation of the proper way to formulate a consequence statement in RIGS GTER PHAM BYED 1, p. 361, fol. 181a4–6: First occurs a statement including the conclusion, such as “Let sound be the subject, it follows that it is not produced, because it is permanent.” After the opponent asks “why?” (*ci’i phyir*) there occurs the statement “What is permanent is not produced, like for instance space; you accept that sound is permanent.”

¹⁹ I take up the issue of the nature of refutation statements at the end of §2.2. “Refutation” in this context refers to an objection attacking the validity of the proponent’s proof. The difficulty is that Phya pa does not give any example when dealing with refutation in MUN SEL section 244. Hence it is difficult to say conclusively what consequences are an alternative to in that case.

²⁰ This is Phya pa’s illustration of a logical reason qua effect in MUN SEL 60b9: *’bras bu ni la la du ba yod pas me yod ces pa lta bu’o //*. The logical reason qua essential property is also illustrated in this form (*rang bzhin ni byas pa’i phyir sgra mi rtag ces pa lta bu’o //*). On the other hand, all examples of reasons qua non-apprehensions put the conclusion in the first place and the logical reason in the second place (for instance: *sa phyogs na snang rung gi bum pa med de ma dmigs pa’i phyir ro*).

²¹ See Hugon 2011 for the details of Phya pa’s theory regarding such proof statements, which are discussed in MUN SEL section 244.

²² The term *rang rgyud* is sometimes applied in the restricted sense of a correct direct proof (*sgrub pa yang dag*) opposed to a fallacious direct proof (*sgrub pa ltar snang*), as for instance in MUN SEL 82b1 and 87b3, but it can also apply to fallacious direct proof statements (cf. MUN SEL 85a9 and 85b2). (See below for the explanation of my use of the term “direct proof.”)

the fact that a consequence is pointed out to “follow” (*thal*).²³

A common English translation of *rang rgyud* in this context is “autonomous proof.” This stresses the fact that the characteristics of the logical reason must be established on their own, in reality, and that this statement leads on its own to an inferential knowledge establishing the thesis. In contrast, consequence statements voice premises that need only be accepted by the opponent. And when they lead to the establishment of a thesis, it is not through the establishment of the characteristics of the logical reason expressed in the statement (e.g., the absence of fire), but through the establishment of the characteristics of the logical reason of the reverse formula (e.g., the presence of smoke).

Rang rgyud statements are also sometimes termed “positive” in opposition to consequences being tagged “negative,” because the first positively establishes a thesis while the second refutes the opponent’s premise(s) by drawing an undesirable conclusion. As I will discuss in more details in §2, such a distinction does not hold for *Phya pa*, since according to him both formulations can (under certain conditions) positively establish the proponent’s thesis. The opposition “positive”/“negative” would only hold with regard to the formulation of logical reason establishing this thesis. Namely, in a *rang rgyud* it is expressed positively (“there is smoke”), while in a consequence it appears in a negative form in the undesirable conclusion “there is no smoke on the pass.” But this conclusion is not even expressed in an actual consequence statement.

I thus prefer to render the term *rang rgyud* as “direct proof” in opposition to the consequence as an “indirect proof,” following a distinction made by *Phya pa* himself when characterizing consequences. Namely, the elements of a *rang rgyud* statement directly express the characteristics of the logical reason that the opponent must remember in order to perform an inference-for-oneself, whereas the elements of a *thal ’gyur* statement enable the opponent to do so indirectly, through several steps of reasoning that will be discussed further in §3.1.²⁴

In addition to referring to the statement of a direct proof, the term *rang rgyud* is also used to designate the “reverse form” of a consequence (*bzlog pa*, Skt. *viparyaya*). This form is obtained by positing the negation of the conclusion of the consequence as a logical reason to establish a thesis which corresponds to the negation of the logical reason of the consequence. Namely, the form

Because S is P, it follows that it is Q	What is P is Q; [according to you] S is P
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is reversed to

Since S is not Q, it is not P	What is notQ is notP; S is notQ
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The reverse form is identical to the alternative formulation of the proof as a direct proof.

1.3 The elements of a consequence

Like a direct proof, a consequence is composed of three interrelated elements. Let us consider their nature in the formula “Because S is P, it follows that it is Q” based on our previous example “Because there is [according to you] no fire on the pass, it follows that there is no smoke”:

- **S** (“the pass”) is called the subject (*chos can*) or basis (*gzhi*).
- **P** (“absence of fire”) is termed the logical reason (*rtags* or *gtan tshigs*) like in a direct proof.
- **Q** (“absence of smoke”) cannot rightly be termed “property to be proven” (*sgrub bya’i chos*) precisely because it is not a property that the proponent wishes to establish for the subject. The proponent only means to indicate that it is a property that the adversary would need to accept if he accepts the logical reason P. Q does not have a specific name, apart from its occasional designation as “what pervades P.” For convenience’s sake I use the expression “derived property.”
- The relation of **P** and **Q** (“where there is no fire there is no smoke”) is discussed in terms of pervasion (*khyab pa*).
- The relation of the logical reason to the subject—**S is P** “there is no fire on the pass”—is sometimes expressed in terms of the reason being a “property of the subject” *phyogs chos* (Skt.

²³ Śākya mchog ldan explains the expression etymologically as “a statement in which an unaccepted property is induced to follow (*thal*)” (RIGS GTER PHAM BYED 1, p. 351, fol. 176a1: *mi ’dod pa’i chos thal bar ’phen pa’i ngag*).

²⁴ The expression *dnogs* to refer to this type of formulation is found for instance in rGyal tshab’s commentary on the RIGS GTER (RIGS GTER RNAM BSHAD 142b2).

pakṣadharmā) like in a direct proof,²⁵ but Phya pa mostly speaks of this feature of the logical reason in terms of the “nature of the logical reason” (*rtags kyi ngo bo*)—or simply of the “logical reason” (*rtags*)—being fulfilled. Unlike in the case of a genuine “property of the subject,” this fulfillment is only a matter of acceptance by the opponent.

- The combination of the subject and the derived property—**S is Q** (“there is no smoke on the pass”)—should likewise not be called “probandum” (*sgrub bya*), even though the term occurs in some discussions. More commonly, Phya pa uses the term *dam bca’*. Although this term means “claim” or “thesis” I prefer to render it in this context as “conclusion.” Indeed, it is not a claim of the proponent, nor is it a claim of the opponent. It is merely what the adversary is forced to derive from the premises if he accepts them.

1.4 Does the speaker always have the choice between the two types of formulation?

Can a proponent who wants to establish a thesis opt for either a direct proof or a consequence in all cases? In theory he does. However, there are circumstances that result in a fallacy of the consequence statement whereas the direct proof would have been faultless. These circumstances pertain to the mental state of the opponent with regard to the proponent’s intended thesis (in our example “there is fire on the pass”).

For a direct-proof statement to be correct, it only matters in this regard that the opponent has not already established (or refuted) this thesis by a means of valid cognition. Otherwise, he would no longer have a “desire to know” (*shes ’dod*) pertaining to the subject and a proof statement would be useless. This requirement also holds for a consequence statement to be pertinent. But here in addition the opponent should accept the opposite of the proponent’s intended thesis. In our example, the opponent should accept that “there is no fire on the pass” although he has not established this fact by a means of valid cognition. In a consequence, the opposite of the proponent’s intended thesis is indeed put forward as a premise from which the undesirable conclusion is derived. If the opponent does not accept this premise, he is not bound to the undesirable conclusion derived from it. In the course of the debate, he can retort to the proponent that the consequence statement is fallacious because its logical reason is not established.

According to Phya pa, the only mental state that guarantees both the “desire to know” and the acceptance of the logical reason of the consequence by the opponent is a mistaken understanding (*log rtags*)²⁶ pertaining to the proponent’s intended thesis. An opponent who mistakenly believes that “there is no fire on the pass” will indeed accept the logical reason of the consequence, but since this mistaken understanding does not amount to a valid cognition, a proof is still needed to lead him to the establishment, by a valid cognition, that “there is fire on the pass.”

The “desire to know” is also preserved in the case of three other mental states: presumption (*yid dpyod*), doubt (*the tshom*), and lack of attention (*blo kha ma phyogs*) with regard to the intended thesis. These three are indeed not valid cognitions, hence a proof is not superfluous. However, an opponent who presumes that there is fire on the pass, who doubts whether or not there is fire, or is not even considering any of these options will not readily subscribe to the premise that “there is no fire on

²⁵ For instance in the discussion relative to the kinds of statements relative to the opponent’s knowledge (see §3.2), and outside the context of the discussion on consequence, when analyzing examples of consequences. gTsang nag pa uses the expressions *phyogs chos* and *sgrub bya* for the qualification of the subject and the derived property of a consequence.

²⁶ I retain here the orthography of the manuscript, which does not systematically distinguish *rtog* and *rtogs*. *rTog* and *rtogs* are more consistently distinguished in classical Tibetan: the first refers to a conceptual cognition, usually negatively connoted as mistaken, the second to a correct understanding, often assimilated to a valid cognition. In view of this distinction, *log rtags* appears as an oxymoron and should thus be taken as mere instance of the older orthography for the more correct *log rtog* (a mistaken notion). But one should consider what the author includes in this category. The way the term is used by Phya pa in the *MUN SEL* and the *’OD ZER* is restricted to cases that can be understood as mistaken conceptual notions. But in *rNgog Lo’s DKA’ GNAS* the term occurs in a triad with *yang dag par rtogs pa* (correct understanding) and *ma rtogs pa* (absence of understanding); there the reading *log rtags* (mistaken understanding) might be intended. In the *sDe dge* version of Sa skya Paṇḍita’s *RIGS GTER* one finds the orthography *log rtog*, but when discussing typologies of mental states the term is used as a synonym of *log shes* and encompasses both conceptual (*rtog pa*) and non-conceptual (*rtog med*) cognitions.

the pass.” These three mental states do not ensure the opponent’s acceptance of the logical reason of the consequence and will prompt the opponent to retort to the opponent that his proof statement is fallacious. Should the opponent be in any of these three states, the proponent is therefore advised to opt for a direct proof.²⁷

2. The function of consequences

Consequences are introduced by *Phya pa* as a way to voice a proof or a refutation in a debate. At first sight, they are thus expected to fulfill the same function as the statement of, respectively, a proof or refutation they are an alternative to. This is true for consequences such as the example discussed in §1.2. But what about consequences whose reversal does not yield a correct direct proof? Indeed, any statement of the form

Because S is P, it follows that it is Q	What is P is Q; [according to you] S is P
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qualifies as a consequence and can in theory be reversed to

Since S is not Q, it is not P	What is notQ is notP; S is notQ
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but the latter does not necessarily constitute a *correct* direct proof.

Let us consider *Phya pa*’s primary classification of consequences:

2.1 *Phya pa*’s primary classification of consequences

Phya pa first distinguishes “genuine consequences” (*thal ’gyur rnal ma*) or simply “consequences” (*thal ’gyur, thal ba*)²⁸ from fallacious consequences (*thal ’gyur ltar snang*). In brief, what characterizes the latter is that the opponent has the possibility to retort, pointing out that he does not accept one or the other premise (i.e., the qualification of the subject by the logical reason or the pervasion of the logical reason by the derived property), or that he actually agrees with the conclusion that follows.

Genuine consequences are further divided into two groups, depending on whether their reverse form constitutes a correct direct proof or not. *Phya pa* speaks of the first as “probans-inducing” (*sgrub byed ’phen pa*)²⁹ or “inducing a correct direct proof” (*rang rgyud ’phen pa*), of the second as “not probans-inducing” or “not inducing a correct direct proof” (*sgrub byed mi ’phen pa/rang rgyud mi ’phen pa*).

A direct proof is correct if it involves a triply characterized logical reason, or in other words, if its logical reason fulfills the two requirements of (I) qualification of the subject and (II) pervasion by the property to be proven, features that must be established by a valid cognition. Comparing the form of the consequence with its reverse form, one can observe that for the reverse form to be a correct direct proof two main conditions must be fulfilled: the pervasion of the consequence must be established by valid cognition, and the conclusion of the consequence (S is Q) must be invalidated by a valid cognition. The first condition ensures that the pervasion of the reverse form (which corresponds to the contrapositive) is established by a valid cognition, the second that the qualification of the subject in the reverse form is established by a valid cognition. A further condition applies in this regard, namely the subject must be established.³⁰ Such consequences constitute proper alternatives to a direct proof (with the restriction pointed out in §1.4). The example dealt with in §1.2 was a consequence of this type.

On the other hand, a consequence such as

Because sound is visible, it follows that it is permanent

addressed to a (strangely philosophically oriented) adversary who holds that what is visible is permanent and that sound is visible but impermanent, is not probans-inducing. It qualifies as genuine because the adversary subscribes to the premises but not to the conclusion that derives from them. But the reverse form

²⁷ See MUN SEL 83a8–9 for an illustration of these three cases for the consequence “Because there is no fire on the pass, it follows that there is no smoke.” In ’OD ZER 145a6–7 the parallel discussion relies on the example “Because [sound] is void of impermanence, it follows that it is void of existence.”

²⁸ See n. 14.

²⁹ The expression possibly comes from Dharmottara’s commentary on the *prasaṅga*-passage of the PVIN.

³⁰ This division of consequences will be dealt with in more details in a sequel to the present paper.

Since sound is impermanent, it is invisible is not correct. Indeed, the logical reason “impermanence” does qualify sound, but it is not pervaded by “invisibility” (for instance, pots are impermanent, but visible). Consequences such as this one, which do not fulfill the conditions for their reverse form to be correct, but do not give the opponent the opportunity to retort, have a distinct place in Phya pa’s classification: although they are genuine, they do not provide a proper alternative to a direct proof.

Let us consider their respective functions. Obviously, fallacious consequences do not fulfill any function. As for genuine consequences, each category carries out different functions:

2.2 The two functions of probans-inducing consequences

Consequences that induce a correct direct proof have two functions: proving and refuting.³¹

a) The function of proving (*sgrub pa’i byed pa*)

A consequence statement of this type fulfills the very same function as the alternative direct proof statement, namely, it proves the speaker’s intended thesis, or in other words, it leads the opponent to an inferential, and therefore valid, understanding of this thesis.³²

Note that the consequence does not prove the conclusion of the consequence (in our previous example, that “there is no smoke on the pass”). This conclusion is not properly speaking “inferred,” because the cognition of something that is not a true state of affairs does not qualify as an “inference” according to the Buddhist epistemologists’ understanding of the term, and also because the conclusion is not grounded in truly established premises.

The probative capacity of a consequence should thus not be evaluated in view of its conclusion, which merely derives, or follows from the premises, but in view of the speaker’s intended thesis (e.g., “there is fire on the pass”).³³ In Phya pa’s words, it fulfills the function of proving what the speaker himself accepts (*rang gi ’dod pa sgrub pa’i byed pa*) and, in concordance with Dharmakīrti’s idea, it does so insofar as it induces a reverse form that amounts to a correct direct proof.

b) The function of refuting (*sun ’byin pa’i byed pa*)

Probans-inducing consequences have an additional function: refuting what the adversary accepts (*pha rol po ’dod pa sun ’byin pa’i byed pa*). As pointed out in §1.4, a condition for positing a genuine consequence is that the adversary has a mistaken understanding (*log rtogs*) pertaining to the speaker’s intended thesis. Thus the function of establishing the proponent’s thesis goes hand in hand with refuting the opponent’s mistaken understanding (also termed “diverging claim” [*mi mthun pa’i dam ’cha’ ba*]) that represents the contrary of the intended thesis.

While consequences that induce a correct direct proof have both the functions of proving and refuting, it is the former that is put to the fore in Phya pa’s alternative characterization of consequences of this type in terms of “proving consequences” (*sgrub pa’i thal ’gyur/thal ba*)³⁴,

³¹ MUN SEL 90b2–4/’OD ZER 146b6–7, and in particular, MUN SEL 90b3–4: *des*_{90b4} *na phyir rgol log rtogs dang ldan pa’i gzhi’ de sgrub pa’i thal ba’i skabs su bab pa yin te / bsgrub bya ma nges pas sgrub pa’i byed pa ’ang ’jug la bsgrub bya las bzlog par dam ’cha’ bas sun ’byin <pa’i byed> pa ’ang ’jug pa’i phyir ro / and ’OD ZER 146b6–7: sgrub pa’i thal ba ni pha rol po’i ’dod pa sun ’byin pa dang rang gi ’dod pa sgrub pa’i*_{146b7} *byed pa gnyis dang ldan pa yin...*

³² Phya pa agrees here with rNgog Lo, who holds that there is only a difference of formulation between a consequence and a direct argument. Both have the same nature insofar as they generate an inference (DKA’ GNAS 385). As I will explain in §3.1, the consequence statement allows the opponent to generate an inference-for-oneself by indicating the features of the logical reason of this inference indirectly, whereas a direct proof indicates them directly.

³³ MUN SEL 88a3ff. For an earlier discussion of the same issues see DKA’ GNAS 381ff.

³⁴ The expression “proving consequence,” prominent in Phya pa’s Madhyamaka works to describe consequences that fulfill the criteria for inducing a correct reverse direct proof, appears to be used interchangeably with the term “probans-inducing” or “inducing a correct direct proof.” For instance in ’OD ZER in the section entitled “definiens of probans-inducing consequences” (146b4 *sgrub byed ’phen pa’i thal ba’i mtshan nyid*) Phya pa introduces both his own definiens (146b6) and that of others (146b4) in terms of “the definiens of a proving consequence” (*sgrub pa’i thal ba’i mtshan nyid*). Śākya mchog ldan notes the equivalence of the two notions for rNgog Lo and his successors in RIGS GTER PHAM BYED 1, p. 382, fol. 190b6–7 (*rngog lo chen po rjes ’brangs dang*_{190b7} *bcas pas sgrub byed ’phen pa’i thal ’gyur yang dag yin na / sgrub pa’i thal ’gyur yin pas khyab par*

opposed to “refuting consequences” (*sun byin pa'i thal 'gyur/thal ba*), i.e., consequences that only refute (see below).

2.3 The function of consequences that do not induce probans

Phya pa is far from being explicit on the function of such consequences, or on the context in which they can be used. However, in both his epistemological and non-epistemological works, one finds many instances of consequences that recognizably belong to this type. When introducing these instances Phya pa does not describe them in terms of “genuine consequences that do not induce a correct direct proof,” but calls them “refuting consequences” (*sun 'byin pa'i thal ba*).³⁵ In one such case (MUN SEL 29b8–9), the consequence is adduced in a section entitled “discarding by means of a consequence” (*thal 'gyur gyis dgag pa*). We may conclude that such a consequence has the function of refuting the opponent’s position.

Phya pa’s successors apply the denomination “refuting consequence” to all consequences that Phya pa classifies as “genuine consequences not inducing a correct reverse proof.” It is likely that this was also Phya’s view, although he does not use this term when dividing genuine consequences in the MUN SEL, and attached it to a single instance of genuine consequence not inducing a correct direct proof in the 'OD ZER.³⁶

2.4 What is refuted by the refuting function?

Genuine consequences that do not induce probans thus do not have the function of proving any thesis, but they have the function of refuting the opponent. We have seen that “refuting” was also among the functions of probans-inducing consequences. However, a difference between the two must be pointed out: in the case of probans-inducing consequences, the refutation applies to a specific view of the opponent, which is the opposite of the proponent’s intended thesis. This tenet of the opponent is posited as a premise in the consequence and is always the premise that corresponds to the qualification of the subject. Indeed, the other premise, the pervasion, must be established by a valid cognition for the reverse form to be correct. But in the case of consequences that do not induce probans, no thesis is established and therefore no specific counter-thesis is refuted. The opponent is certainly refuted in the sense that he must realize that something is wrong with the combination of views that he accepts since they lead to a derived conclusion he does not subscribe to. But the consequence statement does not give an indication as to which of these views should be abandoned. In the example given in §2.1, the consequence “Because sound is visible, it follows that it is permanent” points out the incoherence of the opponent’s position, but since it does not prove

bzhed kyang / rigs gter du de mi rung ste...), and more particularly for Phya pa in RIGS GTER PHAM BYED 1 p. 361, fol. 181a3–4 (*slob dpon rigs pa'i dbang phyug phyug phya pa ni / grangs can de ltar khas len pa la / gang rtag na ma byas pas khyab dper na nam mkha' bzhin / sgra yang* ^{181a4} *rtag par khas blangs so zhes pa tsam zhig 'god par bzhed cing / sgrub byed 'phen pa'i thal 'gyur la sgrub ngag gis khyab par bzhed do //*).

³⁵ For instance, we find consequence statements followed by a comment such as “pervasion is accepted and qualification of the subject is determined by valid cognition.” The fact that pervasion is merely accepted by the opponent but not established in reality prevents the consequence to induce a correct reverse direct proof. See for instance in MUN SEL 29b8–9 the consequence “Because there is in this case no property of the subject, it follows that there is also not what is pervaded by it, namely simple smoke” (*de la phyogs chos med pa'i phyir de'i khyab bya du ba tsam yang med par thal lo*). For other examples where the qualification of the subject is established by a valid cognition and the pervasion is merely accepted by the opponent, which are described by Phya pa as “refuting consequences” (*sun 'byin pa'i thal ba/thal 'gyur*), see 'OD ZER 70a5, 99a3–4, 137b2–3, 138a3–4, 151b1–2 (which notes in addition the feature “conclusion refuted by valid cognition”). There are more examples in Phya pa’s Madhyamaka works, which illustrate other kinds of consequences not inducing a correct direct proof, for instance, those where pervasion is established by valid cognition, qualification of the subject is accepted, but the conclusion is eliminated only by one’s own words. I will deal with this material in a forthcoming part of my study.

³⁶ The term is applied in the 'OD ZER to the case where all the conditions for being a probans-inducing consequence are fulfilled except for the establishment of the negative entailment. 'OD ZER 148a7–8: *rtags khas blangs la rjes 'gro tshad mas nges ste gzhi tshad mas grub pa la dam bca' la tshad mas bsal yang gzhi la bkag pa'i ldog khyab ma nges nas rang rgyud myi 'phen pas sun 'byin pa'i thal ba ste / sgra'i yod* ^{146a8} *pa myi rtag pas stong par khas len pa la sgra'i yod pa myi rtag pas stong bas sgra'i yod pa ma yin par thal zhes pa dang sgra myi rtag pas stong par khas len pa la sgra mi rtag pas stong pas mnyan bya'i yod pas stong par thal zhes pa lta bu'o //*

anything specific, the opponent is left to wonder whether the problem lies in his belief that sound is visible, that sound is impermanent, or that what is visible is permanent.³⁷

2.5 “Proving consequences” and “refuting consequences”: function and/or context?

In the previous discussion, “proving” and “refuting” have been discussed in terms of functions of consequences. The term “proving consequence” (*sgrub pa’i thal ’gyur/thal ba*) was found to be used for probans-inducing consequences that both prove and refute, and “refuting consequence” (*sun byin pa’i thal ’gyur/thal ba*) for consequences that only refute.

Now a passage of the *MUN SEL* in which these two terms occur suggests that they are associated with the contexts of debate consisting in proof—the communication of a proof statement by the proponent—and refutation—the communication by the opponent of an objection to this proof statement.³⁸

Is there a one-to-one correspondence between the understanding of these two terms as referring to functions and as referring to the context of debate, or is there some kind of overlap? According to rNgog Lo, who also uses these two terms in the *DKA’ GNAS*, there is a strict one-to-one correspondence: consequences put forward by the proponent in the context of proof are probative (*sgrub byed*), consequences put forward by the opponent in the context of refutation just point out a mistake in the proponent’s reasoning but are not probative (*sgrub byed ma yin*).³⁹

It is not so clear whether this strict correspondence holds for Phya pa as well. It does not seem problematic for instance to adduce a consequence that both proves and refute in the context of refutation. Such a statement would simply achieve more than it is meant to, but it would fulfill its intended function in this context. Unfortunately, Phya pa does not give any example of refutation when dealing with this step of debate in *MUN SEL* 244, nor in section 245, in which only the title hints to the possibility of presenting a refutation by way of a consequence. It is thus unclear whether the “standard form” of a refutation is a formal direct proof or an informal statement that amounts to naming the fault (e.g., “there is no pervasion”), and whether the alternative formulation in the form of a consequence is restricted to consequences that only refute.

Also, one can observe that the instances identified as “refuting consequences” that are adduced by Phya pa in his works are not all targeting a proponent’s *proof*, but can also simply attack an adversary’s *view*. Their use therefore appears to go beyond the restricted context of refutation understood as a step of debate. It extends to refutation in general.

“Refuting consequences” thus appear as an acceptable tool of argumentation for Phya pa. But we may wonder what would be their status and impact if they are adduced by the *proponent* in a debate. For sure, such a statement would not qualify as a correct proof statement. Indeed when discussing correct proof statements in the context of proof (*MUN SEL* section 244, 82b1) Phya pa lists exclusively “proving consequences” in addition to the acceptable variants of direct proof statements. Also, the proponent would lack the characteristic that defines him as a “proponent” since he would not be able to generate an inference in the opponent’s mind.⁴⁰ However, the opponent would be left unable to retort. Since Phya pa does not discuss points of defeat it remains open whether the opponent’s silence would prevail over the debatable status of the proponent’s statement.⁴¹

³⁷ I will come back in a subsequent part of my study on Phya pa to the similarity between this category of consequences and Candrakīrti’s idea of *prasaṅga*, and on the characterization by Phya pa of the Candrakīrtian *prasaṅga* in terms of *’gal ba sdud pa’i thal ’gyur*.

³⁸ When commenting on the mention of “adversary” (*pha rol po*) in his generic definition of (genuine) consequences, Phya notes that the “adversary” in the case of a proving consequence is the opponent (*phyir rgol*), and that he is the proponent (*rgol ba*) in the case of a refuting consequence. *MUN SEL* 87b1: *pha rol po zhes brjod pa yin te sgrub pa’i thal ba phyir rgol la ’gel la sun ’byin pa’i thal ba rgol ba la ’gel bas de gnyi ga’o /*

³⁹ *DKA’ GNAS* 382–383: *thal bar ’gyur ba rjod pa ni gnyis te / sgrub pa dang sun ’byin pa’o // de la sun ’byin pa ni gtan tshigs kyi skyon tsam rjod pa yin pas sgrub byed ma yin par ’dod pa kho na’o // sgrub pa ni ₃₈₃ sgrub byed yin te / dngos su rjod pa rtags su myi ’dod kyang bzlog pa rtsa ba’i gtan tshigs ’phen pas don gyis na rtags rjod pa’i sgra yin pa’i phyir ro* (This passage is cited in *RIGS GTER PHAM BYED* 1, p. 276, fol. 138b3).

⁴⁰ Cf. *MUN SEL* 81b9. For a discussion of the definition of a “correct proponent” in Tibetan epistemological works see Nemoto 2013.

⁴¹ According to gTsang nag pa, the adversary to whom a consequence that only refutes is addressed would be in a situation of defeat for being unable to retort. But gTsang nag pa does not specify whether the adversary

While the use of the two kinds of genuine consequences in a formal debate remains partially unclear, it is significant with regard to the intellectual context of his time that Phya pa accepts these two kinds. While Tibetan scholars following Candrakīrti were liable to object to the applicability of probans-inducing consequences, there were also some predecessors and contemporaries of Phya pa who did not accept the consequences that do not induce probans as authentic. Phya pa thus rejects in MUN SEL the view of other Tibetan thinkers who hold that there are only “proving consequences.”⁴² The TSHAD BSDUS ascribes this view to “Gangs pa,” presumably Gangs pa she’u Blo gros byang chub, a disciple of rNgog Blo ldan shes rab.⁴³ According to the same text, the same author would also have held the view that “refuting consequences” are only consequences metaphorically speaking (*thal ’gyur btags pa ba yin*), so that there cannot be a common definiens for “proving consequence” and “refuting consequences.”⁴⁴

3. The form of consequence statements

As mentioned in §1.1, one should not confuse Phya pa’s reference to consequences in the form “Because S is P, if follows than it is Q” with the actual form prescribed for consequence statements in a debate. Just like the alternative direct proof statement, a proving consequence statement (a consequence statement that is genuine and induces a correct direct proof) must allow the opponent to perform an inference-for-oneself pertaining to the proponent’s intended thesis. A standard direct proof statement does so by indicating two features of the logical reason from which the thesis can be inferred:

- (I) the qualification of the subject (*phyogs chos*, Skt. *pakṣadharmatā*)
- (II) pervasion by the property to be proven (*khyab pa*, Skt. *vyāpti*).⁴⁵

The verbal expression of these two features works as a reminder for an opponent who had once ascertained these features but has then forgotten them or is not currently paying attention to them. The explicit indication of these two features is a standard form, but it is only one of the eight possible forms that Phya pa accepts as correct direct proof statements.⁴⁶ If the opponent has a vivid awareness of a feature, it does not need to be pointed out to him. On the other hand, if the opponent has not ascertained one or both features, the corresponding indication is replaced by a proof statement enabling the opponent to inferentially ascertain this feature.

A standard direct proof statement indicates the two features of the logical reason directly. For instance, in the direct proof statement

Where there is smoke there is fire, like in the kitchen; there is smoke on the pass, hence...

the first part of the statement directly indicates pervasion (II), the second part directly indicates the qualification of the subject by the logical reason (I).⁴⁷ The alternative standard consequence statement

Where there is no fire there is no smoke, like in the river; [you accept that] there is no fire on the pass, hence....

indicates the same features (which are also the features of the logical reason of the reverse form), but indirectly (*shugs la, don gyis*). How so?

could be the opponent. Cf. BSDUS PA 163a6: *khyad par thal ba brjod pas mi ’dod pa’i thal ba la lan gdab par ma zhus pas pha rol tshar gcad pa’i sgo nas log rtogs sel bar byed pa yin no //*

⁴² MUN SEL 87b5: *kha cig thal ’gyur la sgrub pa’i thal ba las gzhan med do / zhes zer ba /*

⁴³ TSHAD BSDUS 345,4–5: *’di la gangs pa na re thal ’gyur thams cad sgrub byed ’phen pa yin no //*

⁴⁴ TSHAD BSDUS 341,12–342,8: *’di la gangs pa na re thal ’gyur spyi’i mtshan nyid med do //...* Phya pa links the same position with “previous masters” in MUN SEL (87a4 *sngon gyi mkhas pa dag*). The view that there cannot be a common definiens is also mentioned in a marginal note in SGRON MA 62b7; there the author of this view is identified as “Khyung,” i.e., Khyung Rin chen grags, another disciple of rNgog Lo.

⁴⁵ These two features correspond to the three characteristics that define a correct logical reason insofar as the feature of pervasion covers both the characteristic of presence in similar instances only and that of complete absence in dissimilar instances.

⁴⁶ See Hugon 2011: 109–117.

⁴⁷ The order of the two parts could be reversed. However, Phya pa usually gives pervasion first.

3.1 How do the members of the consequence statement indicate the features of the logical reason of the reverse direct proof?

It is clear that the two members of a standard consequence statement directly indicate the features of the qualification of the subject and pervasion of the logical reason of the consequence, and thus lead the adversary to derive the conclusion (“there is no smoke on the pass”). But how do they succeed in indicating the features of qualification of the subject and pervasion of the logical reason of the reverse form, features which the opponent needs to remember in order to perform an inference-for-onself pertaining to the proponent’s intended thesis? For example, how does the statement “Where there is no fire there is no smoke, like in the river; [you accept that] there is no fire on the pass” enable the opponent to remember that (I) smoke qualifies the pass and that (II) smoke is pervaded by fire?

3.1.1 Indication of pervasion (feature II)

There is no major problem as far as the feature of pervasion (II) is concerned. Indeed, pervasion between two elements and its contraposition are logically equivalent, and both can be indicated either via the expression of the positive entailment or the negative entailment. In other words, both “P entails Q” and “nonQ entails nonP” express the same thing: the pervasion of P by Q, or of nonQ by nonP.⁴⁸ “P entails Q” expresses the pervasion of P by Q directly, that of nonQ by nonP indirectly; “nonQ entails nonP” expresses the pervasion of nonQ by nonP directly, that of P by Q indirectly. In our example, the pervasion of smoke by fire can be indicated either by the expression “where there is smoke there is fire” or by the expression “where there is no fire there is no smoke.” Whether he states a direct proof statement or a consequence, the speaker has the choice between these two expressions to indicate the feature of pervasion. Just like a direct proof statement is termed homogeneous or heterogeneous (*chos mthun pa can/mi mthun pa can*, Skt.

sādharmyavat/vaidharmyavat) according to whether it indicates the pervasion via positive entailment or negative entailment, a consequence statement also can take a homogeneous form:

Where there is no fire there is no smoke, like in the river; you accept that there is no fire on the pass, hence....

(*gang me med pa na du ba med de chu klung bzhin la la’ang me med khas blangs pas*)

or an heterogeneous form:

Where there is smoke there is fire, like in the kitchen; you accept that there is no fire on the pass, hence....

(*gang du ba yod pa na me yod de tshang bang bzhin la la me med khas blangs pas*)⁴⁹

3.1.2 Qualification of the subject (feature I)

It is much less evident how the consequence statement indicates the first feature of the logical reason of the reverse proof. As pointed out by Phya pa’s (hypothetical?) opponents, a statement such as “there is no fire on the pass” cannot remind the opponent that “there is smoke on the pass,” nor can the indication that “where there is no fire there is no smoke.” According to Phya pa, it is the combination of the two members of the standard consequence statement that allows to kindle the opponent’s memory or attention with regard to feature I. Upon hearing these two indications, the opponent goes through the following mental process:

- he understands that he should accept the conclusion that follows from the two premises that are directly indicated since that he accepts them (e.g., he should accept that “there is no smoke on the pass”)

- but he understands this conclusion to be erroneous because it is invalidated by a valid cognition (e.g., smoke can be perceived on the pass)

⁴⁸ As discussed in more details in a forthcoming part of my study, the two are always logically equivalent, but there are cases when one of them can be established, but not the other, namely when the pervaded property is not instantiated.

⁴⁹ “Positive entailment” is the form “logical reason entails property to be proven/derived property,” negative entailment is the form “negation of property to be proven/derived property entails negation of logical reason.” The expression of the positive entailment in the direct proof thus corresponds to that of the negative entailment in the consequence, and *vice versa*.

- he understands by implication that the opposite is faultless (e.g., the opposite of the absence of smoke on the pass is the actual state of affairs)
- this amounts to generating the understanding of feature I (e.g., smoke qualifies the pass).⁵⁰

It appears from Phya pa’s explanation that it is actually the “invalidation of the conclusion by a valid cognition” (*dam bca’ la tshad mas gnod pa*) that plays the key-role. Indeed, without this invalidation, the opponent would not consider the derived conclusion to be erroneous and its opposite to be correct. One can understand why some later thinkers claimed that the mention of the invalidation of the conclusion by valid cognition should be made explicit in the consequence statement (see §3.4). But according to Phya pa the two-member statement that expresses the pervasion and the qualification of the subject by the logical reason of the consequence suffices to trigger the thought-process leading to the understanding of feature I. An even though the expression of the qualification of the subject by the logical reason of the consequence (e.g., there is no fire on the pass) may appear unrelated to feature I at first sight, it plays an essential role in bringing about the understanding of this feature.⁵¹

3.2 Variations on the standard form in Phya pa’s model

The standard two-member form advocated by Phya pa allows for a number of variations. Just like direct proof statements (see the beginning of §3), consequence statements must be adapted in relation to the mental state of the opponent. When discussing direct proof statements, Phya pa distinguished three mental states, which he associated with a prescription regarding what should be stated or not in such a case:

feature established by valid cognition and remembered/vividly present to mind	→ no statement required
feature established by valid cognition but forgotten/not paid attention to	→ verbal indication of the feature
feature not established by valid cognition by the opponent	→ embedded proof statement establishing this feature

The mental states to be considered in the case of consequence statements are not the mental states pertaining to the features of the logical reason of the reverse direct proof, but those that pertain to the features of the logical reason of the consequence, features that are adduced as premises for deriving the conclusion. This is because the first step of the mental process that will lead the opponent to an inference-for-oneself is the derivation of the unacceptable conclusion from the premises. The types of mental states to be considered in the case of consequence statements are thus slightly different: The derivation of the conclusion of the consequence requires only the opponent’s acceptance of these premises. However, in order for the consequence to be probans-inducing, its pervasion should in addition be established by a valid cognition. The premise of the qualification of the subject, on the other hand, should not be established by a valid cognition, since it corresponds to the opposite of the proponent’s intended thesis.

Thus the three mental states listed above must still be taken into consideration for the pervasion:

⁵⁰ See notably MUN SEL 88b3–4: *gang me med pa na du ba med de chu klung bzhin la la me med khas blangs pas zhes brjod pa na du ba med pa skyon can khas len dgos par go la des du ba yod pa skyon med du don gyis* ^{88b4} *bstan pas la la du ba yod pa’i rtog pa skyed la / me med la du med kyis khyab pa bstan pa’i shugs la du ba la mes khyab pa la’ang go ba skyed pas rang gi zhe ’dod la me ldan gyi sgrub byed ston pa yin pas sgrub pa’i ngag tsam du ’ang mi ’gal ba’i phyir ro //*

⁵¹ See MUN SEL 89a2–b5 (245.532.16 *dngos kyis phyogs chos brjod pa’i dgos pa*) on the necessity to state the qualification of the subject for the logical reason of the consequence. This “necessity” is not absolute: this statement can be omitted for an opponent who does not need to be reminded that this is what he accepts (see §3.2). Phya pa also argues in this context against some other scholars according to whom the qualification of the subject for the logical reason of the consequence is stated only for the purpose of refuting the opponent. Namely, it points to the mistaken acceptance that leads to the mistaken conclusion. These “other scholars” are, according to the TSHAD BSDUS (351,10), “Byang skyabs”—probably the same as Byang chub skyabs, identified more than 40 times in this text (cf. van der Kuijp 2003: 416)—said here to follow “rGya” (identified more than 60 times), whom van der Kuijp (ibid.) identifies as rGya grags bsod nams, a senior contemporary of Phya pa, rather than Phya pa’s teacher rGya dmar ba Byang chub grags.

pervasion established by valid cognition and remembered/vividly present to mind	→ no statement required
pervasion established by valid cognition but forgotten/not paid attention to	→ verbal indication of the pervasion
pervasion not established by valid cognition by the opponent	→ embedded proof statement establishing this feature

For the qualification of the subject, two mental states must be considered:

qualification of the subject accepted and remembered/vividly present to mind	→ no statement required
qualification of the subject accepted but forgotten/not paid attention to	→ verbal indication of the qualification of the subject

These mental states generate six possible combinations (1–6 in the table below), only four of which (1–4) are taken into consideration by Phya pa. Case no. 5 does not count as a possibility, because when the fulfillment of both features is already vividly acknowledged, there is no need to state anything. The opponent should be able to refute himself and arrive at a correct understanding without the help of the proponent. Case no. 6 is not taken into account either. No explanation is given as to why it is not acceptable, but it is likely that it is in order to avoid ambiguity. Indeed, the consequence statement in such a case would be identical with a direct proof statement establishing the pervasion.⁵²

Since the indication of the pervasion can be done via the mention of positive or negative entailment, the four combinations taken into consideration (1–4) generate six possible forms for consequence statements (i–vi).⁵³

	Qualification of the subject by the logical reason of the consequence	Pervasion	Consequence statement
1	accepted and forgotten	established and forgotten	i Indication of the qualification of the subject and indication of the pervasion by positive entailment (<i>chos mthun pa</i>) ii Indication of the qualification of the subject and indication of the pervasion by negative entailment (<i>chos mi mthun pa</i>)
2	accepted and not forgotten	established and forgotten	iii Only the indication of the pervasion by positive entailment (<i>rjes 'gro tsam</i>) iv (Only) the indication of the pervasion by negative entailment (<i>ldog khyab</i>)
3	accepted and forgotten	established and not forgotten	v Only the indication of the qualification of the subject (<i>rtags tsam</i>)
4	accepted and forgotten	not established by a valid cognition	vi Indication of the qualification of the subject and proof of the pervasion (<i>rtags bkod nas khyab pa sgrub</i>)
(5	accepted and remembered	established and remembered)	-
(6	accepted and remembered	not established)	

⁵² When listing acceptable forms of direct proof statements also Phya pa did not take into account cases that would amount to the mere proof statement of one or the other features. Cf. Hugon 2011: 111, n. 33.

⁵³ See MUN SEL 82b1: *sgrub pa'i thal 'gyur la ni chos mthun pa dang mi mthun pa dang / rjes 'gro brjod pa dang / ldog pa brjod pa dang / phyogs chos brjod pa dang / phyogs chos ston la khyab pa sgrub pa drug ste /*. In the discussion of direct proof statements in 'OD ZER 143a8 Phya pa only mentions that there are six forms of proving consequences. The six forms are discussed in more details with examples in the section of consequences ('OD ZER 148b9–149a5: *sbyor ba'i ngag gi dbye ba*).

By accepting consequence statements consisting in the mere statement of the pervasion (iii and iv), Phya pa opposes a view supported by rNgog Lo.⁵⁴ The latter indeed explains in the DKA' GNAS that the statement of the qualification of the subject is necessary in a consequence; the statement of the pervasion alone is not allowed. So far I understand rNgog Lo's argument in the light of Phya pa's discussion ('OD ZER 149a3–4, MUN SEL 89b5–90a1), this is because otherwise one would not be able to distinguish a heterogeneous direct proof (a) from an homogeneous consequence (b), and *vice versa*:

(a) Where there is no fire there is no smoke, like in the river; *here there is smoke*

(b) Where there is no fire there is no smoke, like in the river; *here there is [according to you] no fire*

According to rNgog Lo, only the statement of the second part (in italics) characterizes the second statement as a consequence. Phya pa argues against this view that it is the context (*skabs*) that allows one to differentiate the two otherwise identical statements. If the context is such that the opponent knows and has not forgotten that there is smoke, it is a direct proof that there is fire by means of the statement of the negative entailment only: the opponent who already knows that there is smoke, once reminded of the pervasion between smoke and fire, will draw the inference that there is fire. If the context is such that the opponent accepts and has not forgotten his acceptance that there is no fire, it is a consequence statement by means of the statement of positive entailment only: the opponent who believes that there is no fire, once reminded of the pervasion of the absence of fire by the absence of smoke, will first draw the conclusion that there is no smoke, and proceed from there to the understanding that there is fire via the mental process described in §3.1.

The statement of the qualification of the subject is thus optional, depending on whether it is needed for the opponent or not. Like in the context of direct proof statements, Phya pa's interpretation of the criterion of "completeness" of the statement is that completeness must be evaluated based on the opponent's understanding and not on a formal basis. Therefore iii, iv and v are "complete" even though they contain only one member. This vision of completeness, which was not followed by Phya pa's students gTsang nag pa and mTshur ston in the case of direct proof statements (cf. Hugon 2011: 127), is also rejected by these authors in the case of consequences.⁵⁵ These authors do not say anything about consequences involving an embedded proof (No. vi), but it is likely that they would not accept them anymore than in the case of direct proof statements (cf. Hugon 2011: 126).

3.3 Should the "absurd conclusion" be stated?

Phya pa's formulation of consequence statements shares with that of direct proof statement the use of the particle *pas/bas*, which I have rendered by "hence..." in my translation of the examples in this paper.⁵⁶ This particle suggests that a conclusion must be derived, without actually stating it explicitly. For instance:

Where there is no fire there is no smoke, like the river; [you accept that] there is no fire on the pass, **hence...**

(*gang me med pa na du ba med de chu klung bzhin la la'ang me med pas zhes*)

Phya pa mentions (MUN SEL 88b7ff.) that some scholars hold that the conclusion must be stated explicitly. One should, for instance, say:

Where there is no fire there is no smoke, like the river; [you accept that] there is no fire on the pass, **hence it follows that there is no smoke.**

(*gang me med pa na du ba med de chu klung bzhin la la'ang me med pas du ba med par thal lo zhes*)

The author of the TSHAD BSDUS (349,4) identifies the holder of this view as "Lo tsa ba," i.e., rNgog Lo. While the latter does not explicitly discuss the necessity to state the conclusion of a consequence in

⁵⁴ MUN SEL 89b5: *gzhan dag thal 'gyur la nges par phyogs chos brjod dgos so zhes zer ba /*. Cf. DKA' GNAS 384–385: *de la phyogs chos ma sbyar bas khyab pa tsam spyir bstan pas ni bye brag la bzlog pa bkag pa ma brjod pas na / brgya la rtogs pa pos spyir bstan pa'i khyab pa bye brag la gzhan bkag pas rtogs kyang bla ste / thal ba brjod pa ma yin no // de'i phyir bzlog pa bkag pa sgras 'phangs pa kho na thal bar 'gyur ba brjod pa yin* ³⁸⁵ *pas phyogs chos rjod pas khyab bo //*

⁵⁵ See BSDUS PA 162b3–4, SGRON MA 65b6–7.

⁵⁶ See Hugon 2011: 108–109 on the adoption of such a formulation in direct proofs by some (but not all) of Phya pa's successors and Sa paṅ's criticism thereof.

his DKA' GNAS, the illustrations of heterogeneous and homogeneous consequences in this work indeed include the statement of the conclusion.⁵⁷ Some of Phya pa's successors also followed this practice (see §3.4).

Phya pa argues against this view that the statement of the conclusion is no more necessary than in the case of a direct proof statement. He follows there Dharmakīrti's final position on the subject, which is to explicitly prohibit the statement of the thesis (or conclusion) on the ground that it does not constitute something that proves (*sādhana*).⁵⁸ Stating the thesis would make the proof statement faulty by including a superfluous element. Phya pa followed this prescription in the case of direct proof statements, and now argues that the same principle applies for consequence statements. The conclusion is understood by implication (*don gyis*), that is, deductively, but should not be stated explicitly. He argues, in addition, that stating the conclusion of the consequence would not be helpful in view of the generation of the inference-for-oneself. A statement such as "hence there is no smoke on the pass" cannot help the opponent remembering the pervasion of smoke by fire or the presence of smoke on the pass.

The omission of the statement of the conclusion has the consequence that the expression "*thal*" does not appear in a consequence statement (as noted in §1.1, it only appears when referring to a consequence statement). From a purely syntactic point of view, the statement of a consequence is identical with the statement of a direct proof.⁵⁹ Will this not create confusion in the debate? Namely, will the opponent be able to figure out whether the cognition he deductively derives is a valid inference or on the contrary an unacceptable conclusion? It may be easy to decide when the conclusion is invalidated by perception (like when the absence of smoke is derived from the premises), but might be problematic when less tangible issues are being discussed. Like in the case of one-member consequence statements (see §3.2) Phya pa argues that the context helps discard any possible confusion: the debaters know what they are debating about and in which circumstances (i.e., to what aim) a statement is put forward. It should therefore be clear to them whether the proof statement that is adduced qualifies as a direct proof statement or as a consequence statement, and they will have no hesitation as to what should be derived as a conclusion and what can be directly or indirectly inferred.

3.4 Should the invalidation of the absurd conclusion be stated?

Phya pa's followers were not unanimous in adopting consequence statements without the mention of the conclusion. They were divided on another issue, namely whether the elimination of the conclusion should be part of the consequence statement.⁶⁰ We have seen indeed (§3.1.2) that it played a major role in establishing the qualification of the subject of the reverse proof. Indicating the invalidation of the conclusion would actually amount to indicating this feature. In Phya pa's opinion there is no need to do so because the standard formulation alone enables the opponent to establish this feature via successive mental steps.

⁵⁷ DKA' GNAS 384: *chos mi 'thun pa yin te / dper na gang na du ba yod pa de na me yod de dper na tshang mang bzhin no // 'di na me med pa'i phyir du ba med par thal lo zhes brjod pa na [...]* *chos 'thun pa yin te / gang na me med pa de na du ba med de / chu lkung bzhin no // 'di na yang me med pas du ba med par thal lo zhes bya bar...* Śākya mchog ldan cites rNgog Lo's illustration of homogenous and heterogenous applications, but adds the mention of the conclusion only in the second. RIGS GTER PHAM BYED 1, p. 360–361, fol. 180b7–181a1: *rngog lo chen po / grangs can sgra rtag par khas len pa la / gang rtag na ma byas pas khyab dper na nam mkha' bzhin sgra yang rtag par grangs can gyis khas blangs so // zhes pa lta bu chos 'thun pa can dang / gang byas na mi rtag pa yin pas khyab dper na bum pa bzhin / sgra ni rtag pa'i phyir ma byas par thal lo zhes pa chos mi 'thun pa can gyis sbyor tshul du bzhed do //*. See also RIGS GTER ROL MTSHO, p. 732, fol. 143b3–4: *lo tsa ba chen po sogs kha cig / thal ba'i chos zlog pas / sgrub pa'i ngag gi phyogs chos nyid 'phen pa yin te / thal ba'i sgrub ngag 'god pa'i tshul 'di ltar / gang me med na du ba med pas khyab / dper na chu klung bzhin du ldan gyi la la'an me yod par khas blangs pas du ba med par thal lo // zhes 'god pas so // zhes zer ro /*

⁵⁸ Cf. PV 4.20, discussed in Tillemans 1984: 90–92, and Tillemans 1991.

⁵⁹ The difference is explicit in some cases where the qualification of the subject is introduced by the words "you accept that..." (*khas blangs te*). But this mention is not a prescribed element of the consequence statement and is not systematically present.

⁶⁰ See RIGS GTER PHAM BYED 1 p. 360–361, fol. 180b7–181a4 for the comparison of the type of statements adopted by rNgog Lo, 'Dan bag pa (spelled Dar 'bag pa), 'Jam pa'i rdo rje (etc.) and Phya pa.

gTsang nag pa as well as his and Phya pa's student mTshur ston follow Phya pa in rejecting the statement of the conclusion⁶¹ but they argue in favor of stating the elimination of the conclusion to help the opponent remember the qualification of the subject of the reverse proof.⁶² They thus advocate a three-member statement such as:

(1) Where there is no fire there is no smoke, like in the kitchen; (2) you accept that there is no fire on the pass, (3) **but it is not the case that there is no smoke.**

(SGRON MA 65b6: *gang me med pa na du ba med pa ste chu klung bzhin no // la la yang me med du khas blangs te du ba ni med pa ma yin no zhes bya ba lta bu'o //*)⁶³

Other scholars adopted consequence statements in which both the conclusion and the elimination of the conclusion are stated explicitly. Śākya mchog ldan attributes such a formulation to Dan 'bag pa (spelled Dar 'bag pa) (one of Phya pa's Eight Great Lions) and his student (gNyal zhig) 'Jam pa'i rdo rje.⁶⁴ For instance:

(1) Permanent is pervaded by not-produced, like the sky; (2) you accept that sound also is permanent, (3) **hence it follows that it is not produced.** (4) **From there, there is elimination by the establishment [of sound] as produced by a valid cognition.**⁶⁵

An analogue formulation was adopted also by Chu mig pa seng ge dpal,⁶⁶ possibly under the influence of gNyal zhig 'Jam pa'i rdo rje via sKyel nag Grags pa seng ge.⁶⁷

⁶¹ BSDUS PA 159b5–160a1, SGRON MA 65a6–7. gTsang nag pa summarizes: *dam bca' mi [read : ni] brjod par bya ba ma yin te dngos kyi bsgrub bya rtogs pa dang bzlog pa rang rgyud 'phen pa'i yan lag ma yin pa'i phyir rang rgyud kyi skabs bzhin no //* (BSDUS PA 162b4).

⁶² BSDUS PA 160a1–2, SGRON MA 65a9–b1. In GSAL BYED gTsang grub rdo rje also argues that the explicit statement of the elimination of the conclusion is necessary to make the opponent remember the qualification of the subject of the direct proof. GSAL BYED 73b24: *rang lugs ni thal ba'i dam bca' ma byas pa la bsal pa brjod pas rang rgyud kyi phyogs chos sgra byas par dran pa yin /* He gives as an example of formal application (73a8): *sgra rtag par khas len pa na gang rtag pa tsam la ma byas pas khyab ste nam mkha' bzhin sgra rtag pa'i phyir ma byas pa'i dam bca' la bsal lo ces pa.*

⁶³ BSDUS PA 159b2: *gang me myed pa de na du ba yang myed pa yin te chu klung bzhin no // la la yang khyod kyi me myed par 'dod de du ba ni me myed [read: ni myed] pa ma yin no zhes bya ba lta bu'o //*

⁶⁴ RIGS GTER PHAM BYED 1, p. 360ff (180b7ff). The connexion between the views of these scholars standing in a master-disciple relationship is suggested also in RIGS GTER PHAM BYED 1 (p. 277, 139a6–b1), where Śākya mchog ldan notes the adoption of the same view pertaining to “proving consequences” by “sMra ba'i seng ge dar (sic) 'bag pa” (=Dan 'bag pa), “slob dpon 'jam pa'i rdo rje” (=gNyal zhig 'Jam pa'i rdo rje), “rgyal ba'i sras po chu mig pa” (=Chu mig pa seng ge dpal) and 'U yug pa in his *Rigs sgrub*. This work by 'U yug pa may have been written when he was one of the so-called nine sons of gNyal zhig (cf. van der Kuijp 1983: 117; 1993: 295); note that 'U yug pa's recently recovered commentary on the RIGS GTER has the title *bsTan bcos tshad ma rigs pa'i gter gyi rgyan rigs pa grub pa*.

Dan 'bag pa is also mentioned together with gNyal zhig (etc.) in RIGS GTER PHAM BYED 1 (p. 361, fol. 181a2: *dar 'bag pa dang / slob dpon 'jam pa'i rdo rje la sogs pa ni*) and RIGS GTER PHAM BYED 2 (p. 458, fol. 229b1: *dan 'bag pa dang / mnyal pa las sogs pa'i bod snga ma rnams*).

⁶⁵ RIGS GTER PHAM BYED 1, p. 360ff., fol. 181a1–2: *dar*_{181a2} *'bag pa dang / slob dpon 'jam pa'i rdo rje la sogs pa ni / gang rtag na ma byas pas khyab dper na nam mkha' bzhin / sgra yang rtag par khas blangs pas ma byas par thal ba las byas par tshad mas grub pas bsal lo zhes bzhed*_{181a3} *la...*

⁶⁶ Cf. RNAM RGYAL A70b9; B62b4 when illustrating homogeneous and heterogeneous consequences: *dang po ni gang mi rtag pas stong ba de byas pas stong namkha' dang 'dra sgra yang mi rtag pas stong bar khas blangs so // sgra la byas par 'dod pa'i dam bca' la sgra byas par tshad mas grub pas bsal lo zhes brjod pa lta bu'o // 2 pa ni gang byas pa la mi rtag pas khyab bum pa dang 'dra sgra ni rtag par khas blangs so //*_{B62b5} *sgra ma byas par 'dod pa'i dam bca' la sgra byas par tshad mas grub pas bsal lo zhes brjod pa lta bu'o //*

⁶⁷ Chu mig pa reports in the colophon of his commentary to the PVIN that he studied with rKyel (for sKyel) grags pa seng ge. The latter is probably identical with sKyel nag Grags pa seng ge, who studied with gNyal/mNyal zhig 'Jam pa'i rdo rje (van der Kuijp 1993: 296); according to the DEB SNGON (p. 407) he was in the last group (*phyi tshar*) of his so-called nine sons. gNyal zhig had been a student of Dan 'bag pa (van der Kuijp 1993: 294) and was abbot of gSang phu for 8 years before rGya 'ching ru ba (also one of the last group of his “nine sons”, who held office for 18 years) and Chu mig pa (DEB SNGON p. 401). For more information on this figure, see Sparham 1996. Chu mig pa is given as gNyal zhig's “grand-disciple” (*nying slob*) by Śākya mchog ldan (see van der Kuijp 1993: 296, n. 43). He must be distinguished from gNyi ba gNyal zhig (full name: rGyal ba ri khrod seng ge) who, according to the DEB SNGON (p. 715) was first a student of Chu mig pa but then became his teacher.

Conclusion — Can “consequences” be assimilated to *reductio ad absurdum*?

Modern scholars often translate *prasaṅga* (resp. *thal 'gyur*) as *reductio ad absurdum*. While there is obviously some family resemblance between the two notions, in the last section of this paper I would like to consider more precisely under which aspects *reductio ad absurdum* does, or does not match Phya pa's understanding and use of consequences. Now that the broad lines of Phya pa's system have been uncovered, we would need, in order to answer this question, a precise model of what *reductio ad absurdum* is. Our task is complicated here by the fact that logic books do not always provide a clear account of its logical structure, and those that do reveal a lack of general agreement regarding this notion. In what follows, I would like to highlight several points, comparing what we have learned of Phya pa's position with the accounts of *reductio ad absurdum* in Kneale&Kneale (1962: 7–9) and in the article “*Reductio ad Absurdum*” by Nicholas Rescher (2005).

(i) Mode of argumentation/mode of expression

There is clearly a common basis between consequence and *reductio ad absurdum* despite the fact that they are integrated in distinct approaches to argumentation in general. Both Kneale&Kneale (1962) and Rescher (2005) discuss *reductio ad absurdum* in terms of a method or *mode of argumentation*.⁶⁸ For Phya pa, *thal 'gyur* is strictly speaking a *mode of expression*. The argument itself, for probans-inducing consequences, is not different from that of a direct proof statement: a thesis is established based on a triply characterized reason. The derivation of an absurd consequence from the denial of this thesis (which corresponds to the opponent's view to be refuted) is only a part of the mental process through which the addressee of the consequence-statement establishes the triply characterized reason. Regardless of the type of statement (direct proof or consequence, homogeneous or heterogeneous), the pattern of inference corresponds to a *modus ponens*.⁶⁹

(ii) Proof and refutation

Kneale&Kneale (1962) traces the Greek origin of *reductio ad absurdum* to Plato's illustration of a dialectical method of argument, namely “the examination of propositions called ‘hypotheses’ by drawing consequences from them” (op. cit., p. 7) and identify this method as an argument-pattern of refutation.⁷⁰ The method indeed does not stop at drawing an absurd conclusion: “If a consequence is unacceptable, the hypothesis from which it is derived must be rejected” (op. cit., p. 7). Kneale&Kneale conclude that *reductio ad absurdum* and *reductio ad impossibile* are primarily

⁶⁸ One should note that this is not the only way that *reductio ad absurdum* is dealt with in the history of logic in the West. It is also characterized as a theorem of propositional logic that can be formalized as: $(\sim p \rightarrow p) \rightarrow p$. Although this view is judged “idiosyncratic” by Rescher, it is widespread in logic texts based on an axiomatic system. Beside Whitehead&Russel's account in *Principia Mathematica* (1910–1913) (mentioned by Rescher) it is also found for instance in Church's *Introduction to Mathematical Logic* (1944) —there the “Law of *reductio ad absurdum*” is given as $p \supset q \supset p \supset \sim q \supset \sim p$, while $p \supset \sim p \supset \sim p$ stands as a “Special law of *reductio ad absurdum*” — or in Kalish&Montague's *Logic: Techniques of Formal Reasoning* (1964) (I thank Jonathan Stoltz for providing me with these references together with a critical appraisal of Rescher's account).

⁶⁹ Namely, the addressee of the statement infers $Q_{(s)}$ (the property to be proven qualifies the subject) from $P_{(s)}$ (the logical reason qualifies the subject) and $P_{(x)} \rightarrow Q_{(x)}$ (the logical reason is pervaded by the property to be proven). In particular in the case of a proof by consequence, the rule of inference that applies is not *modus tollens*. Namely, the addressee does not directly infer $Q_{(s)}$ (equivalent to $\neg A_{(s)}$, $A_{(s)}$ being the opponent's assumption that A qualifies the subject) from $A_{(x)} \rightarrow C_{(x)}$ (the reason in the opponent's assumption is pervaded by the derived property) and $\neg C_{(s)}$ (the negation of the absurd conclusion) but, like in the previous case, by ascertaining $P_{(s)}$ and $P_{(x)} \rightarrow Q_{(x)}$. The idea that *reductio ad absurdum* proceeds via *modus tollens* argumentation is mentioned by Rescher but he himself does not provide further support for this. This characterization is not standard in logic books, but appears to be suggested for instance in Kneale&Kneale's description of the argument-pattern of refutation as following the logical schema “If P then Q; but not-Q; therefore not-P” (op. cit., p. 7).

⁷⁰ They note that this method might have been suggested to Plato by Socrates' practice or Zeno's use of *reductio ad impossibile*, in which incompatible consequences are drawn from the hypothesis. But, as Kneale&Kneale note, although Plato may not have made this distinction, “the Socratic elenchus differs from the Zenonian in that the consequences drawn from the hypothesis need not be self-contradictory but may on occasions be simply false” (op. cit., p. 9). The “hypothesis” may be a pure hypothesis (for Zeno) or an opponent's opinion (for Socrates).

techniques used for the purpose of refutation: “It is clear that, in general, this procedure can lead only to negative results” (op. cit., p. 7). But they also mention some facts that hint to the use of this procedure to defend or establish a thesis.⁷¹

The two uses of this mode of argumentation also come to the fore without a strict distinction in Rescher’s account. *Reductio ad absurdum* is described as a “process of refutation on grounds that absurd – and patently untenable consequences would ensue from accepting the item at issue.” But it is also a “mode of argumentation that seeks to establish a contention by deriving an absurdity from its denial, thus arguing that a thesis must be accepted because its rejection would be untenable.” As such *reductio ad absurdum* is a “special case of demonstrative reasoning” also called “indirect proof.”

Kneale&Kneale and Rescher thus do not strictly distinguish between the use of *reductio ad absurdum* for the sake of refutation or for the sake of proof. This lack of distinction is matched in Phya pa’s category of probans-inducing consequences, since such consequences have both the function of proving the proponent’s thesis and refuting the opponent’s view. But this is, in Phya pa’s system, only one of the two types of genuine consequences that are taken into account. There are also consequences that only refute, not because the proponent only intends to refute and does not want to claim that anything is established positively, but because they are not probans-inducing, and therefore cannot prove anything.

(iii) Refutation but no proof

Kneale&Kneale do not mention a case of argumentation by *reductio ad absurdum* used for the sake of refutation that cannot also count as a proof. Rescher singles out one instance where the *reductio ad absurdum* is not accepted as a means of proof. Namely, the so-called intuitionist mathematical school does not allow *reductio ad absurdum* argumentation for proofs of existence: a concrete instance must be provided. More generally, one can say that in intuitionist logic, *reductio ad absurdum* can only lead to a negation (whether it is conceived as the refutation of an assumption, or as the proof of a negative thesis), because the law of excluded middle is not accepted, with the consequence that a double negation does not imply an affirmation.⁷²

An inquiry into the possible connexion between intuitionism and Madhyamaka is beyond the scope of this paper. And while a similar issue is raised by Phya pa in the case of unestablished subjects, it does not directly concern the equivalence between the negation of the opponent’s view and the establishment of a thesis.⁷³ In Phya pa’s system, consequences that are not probans-inducing do not

⁷¹ They mention for instance that “In his *Parmenides* Plato makes Zeno claim to have written a book in which he defends the monism of Parmenides by drawing out the absurd consequences of the supposition that there is plurality” (op. cit., p. 7), or the proof of the incommensurability of the diagonal with the side of a square by the Pythagoreans (op. cit., p. 8). They also mention a controversial passage of the *Republic* in which “Plato seems to have meant something more precise by ‘dialectic’, namely a method of argument involving refutation but leading eventually to positive results of high generality” (op. cit., p. 9).

⁷² Cf. Moschovakis 2010: “Intuitionistically, *Reductio ad absurdum* only proves *negative* statements, since $\neg\neg A \rightarrow A$ does not hold in general. (If it did, LEM would follow by *modus ponens* from the intuitionistically provable $\neg\neg(A \vee \neg A)$.” (LEM=Law of Excluded Middle). The issue with proofs of existence follows from this principle. Namely, $\exists xA(x)$ cannot be derived from $\neg\forall x\neg A(x)$. Thus, by drawing an absurd or false conclusion from $\forall x\neg A(x)$ one cannot prove $\exists xA(x)$.

⁷³ The issue for Phya pa is raised with regard to the positive establishment of the qualification of the subject in the reverse proof. When the subject is established, the negation of the absurd conclusion of the consequence amounts to the positive establishment of the qualification of the subject in the reverse proof (in our stock-example, the negation of the absurd conclusion that “there is no smoke on the hill” is tantamount to the establishment that “there is smoke on the hill”). But when the subject is not established, according to Phya pa, one may be able to negate a characterization applying to it (in particular, one can negate the absurd conclusion), but one cannot establish the opposite characterization in a positive way, which is in his opinion required for the qualification of the subject to be established in the reverse proof. This does not imply that Phya pa is adopting an intuitionist approach: for him the problem is not that the subject represents an infinite collection, only part of which can be asserted to be qualified or not by a property, but that the subject, being unestablished, is outside the scope of objects to which valid means of cognition apply. Negation of the qualification by a given property can be achieved indirectly in some cases, but not the affirmation of a property. I will come back in more details to this issue in a subsequent part of my study of Phya pa’s views on consequences.

even prove a negative thesis—they do not prove anything.

(iv) Refutation of a single tenet/refutation of the coherence of a set of tenets

A further point regarding consequences that are not probans-inducing is that for Phya pa (cf. §2.4) their refuting function bears on the coherence of a set of tenets accepted by the opponent, but does not single out one of them as the assumption that is to be abandoned. Rescher mentions that *reductio ad absurdum* can be used for “doctrinal annihilation,” namely, to prove that a set of statements are collectively inconsistent. But his illustration of this use singles out one supposition which, when added to other premises, result in a self-contradiction, from which the refutation of *this* supposition follows. He does not evoke the possibility that the derivation of the absurd conclusion from the opponent’s assumption could rely on a connexion (or a chain of connexions) between the antecedent and the consequent which is only accepted by the opponent, or that the conclusion itself could be “absurd” only from the point of view of the opponent.

In spite of the similarity between the notion of consequence and *reductio ad absurdum*, there is a specificity in the approach of Tibetan logicians such as Phya pa to this topic: they do not apprehend it as a logical form, but focus rather on the elements of the verbal statement and on the mental process this statement generates in the mind of the opponent. In this regard, compared to direct proofs, consequences involve a verbal difference and additional elements in the mental process of the addressee, but are not strictly speaking different arguments. In addition, an equivalent to the full range of Phya pa’s category of consequences that do not induce probans is not commonly found in Western discussions on *reductio ad absurdum*, which focus on *reductio* as an indirect proof. The conclusion I would like to draw from the above remarks is not that we should refrain from using the term *reductio ad absurdum* when speaking about Indian theories of *prasaṅga* and Tibetan theories of *thal ’gyur*. There is a sufficient similarity between them, and sufficient latitude in the various ways *reductio ad absurdum* is conceived in Western logic (probably as many as there are theories of *prasaṅga* and of *thal ’gyur* among Indian and Tibetan thinkers) for this term to be applicable without being *a priori* loaded with definite connotations. If one should welcome the possibility to facilitate the dialogue with scholars outside the field of Indology or Tibetology in this way, it should be clear however that the mere application of the term cannot be a substitute for a detailed investigation spelling out the views of individual authors and doing justice to the specific contribution of Buddhist thinkers within their own intellectual environment as well as in the broader context of world philosophy.⁷⁴

Acknowledgments

Work on this paper has been generously supported by the Austrian Science Fund (FWF) in the context of the FWF-Project P23422-G15 “Early bKa’ gdams pa scholasticism.” I am very grateful to Jonathan Stoltz for his insightful remarks on an earlier version of the last part of this paper. My thanks also to Ernst Steinkellner for his useful comments.

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⁷⁴ See in connexion to this Tillemans’ remarks on the use of the word “syllogism” to speak of the Buddhist inference-for-others (*parārthānumāna*): “[...] if we satisfy ourselves with superficial similarities we blur the philosophically interesting point that Buddhist logic is *sui generis*. Thus we preclude meaningful, informed attempts at comparative philosophy.” (Tillemans 1991: 147).

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